IS THE CLUSTER TEMPERATURE FUNCTION A RELIABLE TEST FOR Ω_0?

SERGIO COLAFRANCESCO

Osservatorio Astronomico di Roma

Via dell'Osservatorio 2, I-00040 Monteporzio, Italy

PASQUALE MAZZOTTA

Dipartimento di Fisica, Università di Roma "Tor Vergata"

Via della Ricerca Scientifica 1, I-00133 Roma, Italy

and

NICOLA VITTORIO

Dipartimento di Fisica, Università di Roma "Tor Vergata"

Via della Ricerca Scientifica 1, I-00133 Roma, Italy

Received ______________________; accepted ______________________

ABSTRACT

We discuss the evolution of the cluster temperature function (TF) in different scenarios for structure formation. We use the commonly adopted procedure of fitting the model parameters to the local TF data, finding the best fit values and, most of all, the associated statistical errors. These errors yield an uncertainty in the prediction of the TF evolution. We conclude that, at the moment, observations of cluster temperatures at $z \lesssim 0.5$ could provide only a weak test for Ω_0.

Subject headings: cosmology: theory – cosmology: observations – cosmology: dark matter – galaxies: clusters: evolution – intergalactic medium

1. Introduction

The gravitation instability in a flat universe seems to provide a reasonable scenario for structure formation. COBE/DMR (Smoot *et al.* 1992; Gorski *et al.* 1994; Banday *et al.* 1994) and the more recent CMB anisotropy experiments at degree angular scales (see *e.g.* de Bernardis *et al.* 1996 for a discussion) unvealed the presence of linear density fuctuations at recombination, in quite a good agreement with the theoretical predictions of dark matter dominated models. However, the non-linear evolution of these fluctuations and the actual formation of objects like galaxies or clusters of galaxies is far from being understood. Being the latest structures formed by the present time via a dissipationless collapse, clusters of galaxies play a crucial role in linking the linear and non-linear regimes of the gravitational instability theory. So, from one hand the Press & Schechter (1974, hereafter P&S; see also Bond *et al.* 1991) theory allows to predict the abundance and evolution of clusters of galaxies in a simple and semi-analytical way, by using the linear theory of structure formation. On the other hand, the available surveys of clusters of galaxies in the X-rays allow to construct observables like the local luminosity (see, *e.g.* , Kowalski et al. 1984 and Ebeling *et al.* 1997), and temperature (Henry & Arnaud 1991, hereafter H&A; Edge et al. 1990) functions.

Theoretical predictions for the X-Ray Luminosity Function (XRLF) heavily rely upon assumptions on the amount, the distribution and the evolution of the intra-cluster (IC) gas. Observations seem to indicate that the IC gas is a fraction between 10 and perhaps 30% of the cluster virial mass (White *et al.* 1993, Cirimele, Nesci & Trevese 1997). Moreover, even for virialized clusters the physics behind the formation of a core in the gas distribution is not known. Finally, there have been claims that the IC gas rapidely decrease for redshifts larger than 0.3 (Gioia *et al.* 1990, Henry *et al.* 1992, Cavaliere & Colafrancesco 1988, Kaiser 1991), although it is very hard at the moment to draw definitive conclusions on

this issue. In fact, it seems that the amount of XRLF evolution at $z \lesssim 0.5$ is somewhat less (Ebeling *et al.* 1997, Nichol *et al.* 1996, Romer *et al.* 1997) than suggested by the Extended Medium Sensitivity Survey (Gioia *et al.* 1990, Henry *et al.* 1992). Because of these uncertainties, it has been widely argued that the cluster temperature function (TF) is a more reasonable quantity to work with (H&A, Kaiser 1991). The basic point is that the temperature of the IC gas in hydrostatic equilibrium with the potential well of a virialized cluster depends only on the cluster virial mass.

In spite of this simplification, it turns out that in dark matter, COBE/DMR normalized models the theoretical TF is not consistent with the local data (Eke *et al.* 1996, hereafter Eke *et al.*). Thus, the usual procedure is to tune the model parameters to fit the local TF and, then, to make predictions for other observables such as the cluster number counts and the Sunyaev-Zel'dovich effect (Barbosa *et al.* 1996, Eke *et al.*).

Following this procedure, it has been shown that: *i)* the TF evolution in flat and open cosmologies is drastically different; *ii)* observations of cluster temperatures at redshifts $z \gtrsim 0.3$ can strongly constrain Ω_0 (Hattori and Matsuzawa 1995, hereafter H&M; Eke *et al.* 1996, Barbosa *et al.* 1996, Oukbir and Blanchard 1992, Oukbir and Blanchard 1997). While we fully agree with point *i)*, we believe that point *ii)* requires a more carefull investigation.

Thus, the purpose of this paper is to show that: *i)* the amount of TF evolution is heavily determined by the fit to the local data, and is not a self-consistent prediction of the theoretical models; *ii)* the intrinsic statistical uncertainty of this fit smeares out the difference between the low density and $\Omega_0 = 1$ model predictions for the TF evolution.

The plan of the paper is as follows. In Sect.2 we review the basic steps behind the theoretical calculations. In Sect.3 we discuss in a simple semi-analytical way what determines the TF evolution. In Sect.4 we show the constraints on the model parameters set by the fit to the local TF. In Sect.5 we present detailed numerical predictions of the TF

evolution in different structure formation models, including Cold Dark Matter (hereafter CDM) cosmologies (flat, open and vacuum dominated). Finally, in Sect.6 we summarize our main findings.

2. The cluster TF

The cluster TF is defined as

$$N(T,z) = \mathcal{N}(M,z)\frac{dM}{dT} \tag{1}$$

where $\mathcal{N}(M,z)$ is the cluster mass function (MF). The latter is usually derived by the P&S theory:

$$\mathcal{N}(M,z) = \sqrt{\frac{2}{\pi}}\frac{\rho}{M^2}\frac{\delta_v}{\sigma}\frac{dln\sigma}{dlnM}\exp[-\delta_v^2/2\sigma^2]\,, \tag{2}$$

where ρ is the comoving background density, M is the total cluster mass and δ_v is the linear density contrast of a perturbation that virializes at z. The variance of the (linear) density fluctuation field at the scale $R = (3M/4\pi\rho)^{1/3}$ and redshift z is given by the standard relation (see *e.g.* Peebles 1980):

$$\sigma^2(R,z) = \frac{D^2(\Omega_0,z)}{2\pi^2}\int k^2 dk P(k)\Big[\frac{3j_1(kR)}{kR}\Big]^2 \tag{3}$$

where $D(\Omega_0,z)$ is the growth factor of linear density fluctuations in a given cosmology, and j_1 is a spherical Bessel function. If we normalize the matter power spectrum, $P(k)$, by requiring $\sigma(8\ h^{-1}\ \mathrm{Mpc},0) = b^{-1}$, then the MF depends on the product $b\delta_v$, and not separately on δ_v and b.

While the cluster MF gives the mass and redshift distribution of a population of evolving clusters, the Jacobian dM/dT describes the physical properties of the single cluster. Under the standard assumption of the IC gas in hydrostatic equilibrium with the potential well of a spherically simmetric, virialized cluster, the IC gas temperature–cluster

mass relation is easily obtained by applying the virial theorem: $T = -\mu m_p U/(3K_B M)$, where $\mu = 0.62$ is the mean molecular weight (corresponding to a Hydrogen mass fraction of 0.69), m_p is the proton mass, K_B is the Boltzmann constant and U is the cluster potential energy. If the cluster is assumed to be uniform, $U = -(3/5)GM^2/R_v$ and $T = T^{(u)} \equiv (1/5)(\mu m_p/K_B)GM/R_v$, where $R_v = [3M/(4\pi\rho\Delta)]^{1/3}/(1+z)$ is the cluster virial radius and $\Delta(\Omega_0, z) = 18\pi^2/[\Omega_0(H_0 t)^2(1+z)^3]$ is the non linear density contrast of a cluster that virializes at z in a $\Omega_0 \leq 1$ cosmological model [in flat, vacuum dominated low density models Δ has not an analytical expression (see, *e.g.* Colafrancesco *et al.* 1997 and references therein)].

We also relax the assumption of uniformity by considering a 3-D gas density profile:

$$n(r) = n_c \left[1 + \left(\frac{r}{r_c}\right)^2\right]^{-3\beta/2} \tag{4}$$

where n_c is the central electron density and r_c is a core radius. The mass within the outer (proper) radius, taken as $R = p r_c$, is $M(p) = 3M_c \omega(p, \beta)$, where $\omega(p, \beta) = \int_0^p t^2 dt/(1+t^2)^{3\beta/2}$, $t \equiv r/r_c$, $M_c = (4\pi/3) r_c^3 \rho_c$, and ρ_c is the central total mass density of the cluster. Because of the assumed profile, the ratio between the central and mean mass density of the cluster is $\rho_c/(\rho\Delta) = p^3/3\omega(p, \beta)$. Then, $r_c = R_v/p$, or, equivalently,

$$r_c(\Omega_0, M, z) = \frac{1.29\ h^{-1}\ \mathrm{Mpc}}{p} \left[\frac{M}{M_{15}} \cdot \frac{\Delta(1,0)}{\Omega_0 \Delta(\Omega_0, z)}\right]^{1/3} \frac{1}{1+z} \tag{5}$$

where $M_{15} = 10^{15}\,\mathrm{h}^{-1}\ \mathrm{M}_\odot$. In this case, $U = -(GM^2/r_c)\psi/\omega^2$ and $T = 5p\psi T^{(u)}/(3\omega^2)$, where $\psi(p, \beta) = \int_0^p s ds/(1+s^2)^{3\beta/2} \int_0^s t^2 dt/(1+t^2)^{3\beta/2}$. Hereafter we fix $p = 10$ to recover r_c values consistent with the observations (see *e.g.* Henriksen and Mushotsky 1985, Jones and Forman, 1992).

At this point, the cluster TF is fully determined as we know

$$M(T, z) \propto \frac{T^{3/2}}{\sqrt{\Omega_0 \Delta_v(\Omega_0, z)}} \frac{1}{(1+z)^{3/2}} \frac{1}{h} \tag{6}$$

and $dM/dT = 3M/2T$.

3. The TF Evolution: an analytical approach

In this Section we restrict ourselves to the simple case of power–law power spectra (hereafter PLPS). This allows us to write down an explicit expression for the TF, by using the results of Sect.2. In particular, the $M(T,t)$ relation provides for the mass variance (in this case $\propto M^{-\alpha}$) the following expression:

$$\sigma[M(T,t),t] = \frac{1}{b}\Big(\frac{T}{T_0}\Big)^{-3\alpha/2}\Big(\frac{t}{t_0}\Big)^{-\alpha} D(\Omega_0,t) \tag{7}$$

where $\alpha = (n+3)/6$ and n is the spectral index. Here T_0 is the present IC gas temperature of a cluster of mass M_0, corresponding to the normalization scale of $8h^{-1}Mpc$. Because of the use of T and t as independent variables, the time dependence of σ is different from the standard one: the extra factor, $(t/t_0)^{-\alpha}$, takes into account the fact that a cluster with a given temperature at $t < t_0$ is less massive than a cluster with the same temperature at t_0. Along the same line, it is easy to verify that:

$$\frac{1}{M^2}\frac{dM}{dT} = \frac{3}{2}\frac{1}{M_0 T_0}\Big(\frac{T}{T_0}\Big)^{-5/2}\Big(\frac{t}{t_0}\Big)^{-1} \tag{8}$$

Equations (7) and (8) allow to write the TF as follows:

$$N(T,z) = C\Big(\frac{T}{T_0}\Big)^{(-5+3\alpha)/2}\Big(\frac{t}{t_0}\Big)^{-1+\alpha}\frac{1}{D}\exp\Big[-\frac{(\delta_v b)^2}{2}\Big(\frac{T}{T_0}\Big)^{3\alpha}\Big(\frac{t}{t_0}\Big)^{2\alpha}\frac{1}{D^2}\Big] \tag{9}$$

where C is a constant which depends on Ω_0, h^3, and $\delta_v b$. For didactic purposes, it is worth to derivate Eq.(9) w.r.t. time: this will help to understand which quantities define the amount of TF evolution. It is immediate to verify that:

$$\frac{dlnN}{dt} = -\frac{1}{t} - \frac{1}{\tau_-} + \frac{1}{\tau_+} \tag{10}$$

where $1/\tau_- = \dot{D}/D - \alpha/t$ and $1/\tau_+ = (\delta_v b)^2 (T/T_0)^{3\alpha}(t/t_0)^{2\alpha}/(D^2\tau_-)$.

The amount and kind (positive or negative) of TF evolution is set by the competition of these three time scales: the cosmic time t, τ_- and τ_+. When either t and/or τ_- are shorter

than τ_+, then the TF evolution is negative. When τ_+ is shorter than either t or τ_-, then the TF evolution is positive. At a given cosmic time, τ_- depends on the chosen cosmology through the logarithmic derivative of the growth factor. In principle τ_- depends also on the shape of the power spectrum: in practise, for $\Omega_0 \leq 1$ models with no cosmological costant, $\alpha/t < \dot{D}/D$. The other time scale, τ_+, at a given time and temperature depends weakly on the spectral index n and quadratically on the $\delta_v b$ parameter. We call the reader's attention on this point. In fact, at least for PLPS, both n and $\delta_v b$ are determined by fitting the theoretical prediction for the local TF to the existing data. Thus, for a given cosmology, the TF evolution can be positive or negative, depending on the derived values for n and, most of all, for $\delta_v b$. So, while fitting the theoretical TF to its local, observed values seems "the best one can do" in the absence of a more refined theory for cluster formation and evolution, we can not forget that the amount and kind of the TF evolution is already determined just by this fitting procedure.

Similar conclusions are of course reached if we use redshift instead of cosmic time as an independent variable. In this case we have

$$\frac{dN(T,z)}{dz} = -\frac{dt}{dz} N(T,z) \frac{d\ln N(T,t)}{dt} \tag{11}$$

where, we remind it, $N(T,t)$ is itself proportional to the $\delta_v b$, while dt/dz depends only on the chosen cosmology.

The purpose of this Section was to show qualitatively the central role that $\delta_v b$ plays (together with n, at least in the PLPS models) in defining the TF evolution for a given cosmological model. Thus, we believe that it is important not only to estimate best fit values, but also the statistical uncertainties associated with them. This is the goal of the next Section.

4. Fit to the local FT

In this Section we derive some model parameters by fitting the TF theoretical predictions to the H&A data. We use flat and open models where density fluctuations are described by PLPS, and CDM cosmologies, either flat or open or vacuum dominated.

We use the following mass-temperature relation [see Eq.(6)]:

$$M = M_{15}\Big(\frac{T}{T_{15}}\Big)^{3/2}\Big(\frac{t}{t_0}\Big) \qquad (12)$$

Accordingly to the uniform cluster model described in Sect.2, a cluster of mass M_{15} has a temperature $T_{15} = 4.3$ keV. However, to compare our results with those of H&A and H&M we will use in the following $T_{15} = 6.4$ keV. This value was choosen by H&A and H&M to be consistent with the numerical results of Evrard (1991, hereafter Evrard), who also found from his simulations a scatter of $\approx 10\%$ in the M-T relation.

Consistently with the observations, we express the TF as the number of cluster per $(h^{-1}Mpc)^3$ per keV. Then, the PLPS model predictions for the TF are independent of h [cf. Eq.(9)]. Thus, we consider as free parameters both $\delta_v b$ and the spectral index n. The best fit values are shown in Table 1, together with the uncertainty in the fit at the 68.3% and, in parenthesis, at the 90% confidence level. Our uncertainties are smaller than those quoted by H&A. This is because we did not consider (as H&A did) the 10% uncertainty on the $M-T$ relation derived from Evrard simulations. However, even at the level we quoted them, these uncertainties have serious impact on the model predictions as we will show in the next Section.

Note that $\delta_v = 2.2$ is the density contrast at virialization extrapolated from the linear theory. This numerical value changes very little varying Ω_0 and z. If we assume that all clusters are virialized, then the value of the biasing parameter is of the order of unity for $\Omega_0 = 1$, but less than unity (antibias) for low density universes. This is due to the following

fact. The mass contained in a 8 h^{-1} Mpc sphere is $M_0 = 0.63\Omega_0 M_{15}$. According to Eq.(12), the collapse of such a sphere will results in clusters with temperature of 4.7, 1.6 and 1.0 keV for $\Omega_0 = 1$, 0.2 and 0.1, respectively. In order to form Coma-like clusters (*i.e.* with $M \simeq M_{15}$ and $T \simeq 6$keV) in a low density universe, we have to assume that those density fluctuations that generate clusters have amplitudes larger than the mass density field: in other words, clusters must be antibiased. Our values of b are quite consistent with the scaling found by White *et al.* (1993): $b = 1/\sigma(8\ h^{-1}\ \mathrm{Mpc}) \simeq 1.75\Omega_0^{0.56}$. In fact, we find $b = 1.7\Omega_0^{0.58}$ if we choose $\delta_v = 1.68$ (if we identify clusters as collapsed rather than virialized objects). The small difference between our result and that of White *et al.* (1993) depends on the different datasets used for the cluster abundance.

For CDM cosmologies, we use both the uniform cluster and the β-profile models. If we use the uniform cluster model, with fixed Ω_0 and h, we get the same results as Bartlett and Silk (1993). If we assume a β-profile for the IC gas distribution, then $T_{15} = 5.8$ keV and 7.8 keV for $\beta = 2/3$ and 0.93, respectively. The latter temperature value is the one found by Eke *et al.* analyzing numerical simulations, which they show to be quite consistent with the P&S predictions.

At variance with the PLPS models, in the CDM scenario h contributes, together with Ω_0, to define the shape of the power spectrum. So, keeping the β-profile for the IC gas distribution with $\beta = 2/3$ and assuming $n = 1$, we fit the local FT data by considering as free parameters $\delta_v b$, Ω_0 and h. We get the following best fit values: $\Omega_0 = 0.4^{+0.10(+0.15)}_{-0.11(-0.14)}$, $h = 0.6^{+0.4(+0.4)}_{-0.1(-0.1)}$ and $\delta_v b = 1.6^{+0.05(+0.14)}_{-0.13(-0.19)}$. Note that we force h to be in the range: $0.5 \leq h \leq 1$.

Although a reasonable fit is obtained in a quite broad region of the parameter space, it must be stressed that, as already found by H&A, the standard flat CDM model with $h = 0.5$ does not provide a good fit to the TF data: for such a model the reduced χ^2 is 1.8 (see

Table 2). Our results for CDM models are also consistent, within the quoted uncertainties, with those found by Viana and Liddle (1996), using $N(7\mathrm{keV}, z=0)$ to derive the amplitude of the fluctuation spectrum. The corresponding uncertainties are of the order of +35% and −25% at 95% confidence level for low density, vacuum dominated CDM models (hereafter CDM+Λ), quite independently on the assumed modelling of the TF.

5. Results

Now we can use the best fit values to predict the cluster TF evolution. Let us first consider PLPS models. With the best fit values of n and $\delta_v b$, we find that the TF evolution is quite different in open and flat models: the abundance of clusters of a given temperature predicted in an $\Omega_0 = 0.1$ universe is substantially larger than in the flat model, a result already obtained by H&M. However, this does not necessarely imply that we can estimate Ω_0 by using data on the TF evolution. We have first to quantify the probability of concluding that the universe is low (high) density when it is actually high (low) density. Let us assume that the TF depends only on b. Then, the uncertainty on $N(T,z)$ due to the uncertainty on b (derived from fitting to the local TF) writes as:

$$\frac{\Delta N}{N} = \frac{1}{N}\left|\frac{\partial N}{\partial b}\right|\Delta b \qquad (13)$$

The TF can be written as $N(T,z) \propto g\exp[-0.5g^2]$ where for PLPS $g = [\delta_v b/D(t)](M/M_0)^\alpha$. Because of our mass-temperature relation [*c.f.* Eq.(12)] we can write (for $\Omega_0 \leq 1$ and $\Lambda = 0$)

$$g = \frac{\delta_v b}{D(t)}\left(\frac{T}{T_0}\right)^{3\alpha/2}\left(\frac{t}{t_0}\right)^{\alpha} \qquad (14)$$

and, if we further assume $\Omega_0 = 1$,

$$g = \delta_v b(1+z)^{1-3\alpha/2}\left(\frac{T}{T_0}\right)^{3\alpha/2} \qquad (15)$$

So we have:

$$\left(\frac{\Delta N}{N}\right) = \left(\frac{\Delta b}{b}\right) \times \left|\left[1 - (\delta_v b)^2(1+z)^{2-3\alpha}\left(\frac{T}{T_0}\right)^{3\alpha}\right]\right| \qquad (16)$$

Now, for $\Omega_0 = 1$ and PLPS, the best fit values are $n = -1.8$ and $\delta_v b = 2.8$, the latter with an uncertainty $\Delta b/b \simeq 9\%$ at the 90% confidence level. This uncertainty yields an uncertainty on $N(T,z)$ which increases either with T and/or with z, as $2 > 3\alpha$. Moreover, in an $\Omega_0 = 1$ universe, clusters with $M = M_0$ have a temperature $T_0 = 6.4(0.63)^{2/3}$keV. As a result, for $\Omega_0 = 1$ and $n = -1.8$ we get:

$$
\begin{aligned}
\frac{\Delta N}{N}(T, z=0.0) &= \left(\frac{\Delta b}{b}\right) \times \left|[1 - 3.1T^{0.6}(keV)]\right| \\
\frac{\Delta N}{N}(T, z=0.5) &= \left(\frac{\Delta b}{b}\right) \times \left|[1 - 5.5T^{0.6}(keV)]\right| \\
\frac{\Delta N}{N}(T, z=1.0) &= \left(\frac{\Delta b}{b}\right) \times \left|[1 - 8.2T^{0.6}(keV)]\right|
\end{aligned}
\tag{17}
$$

From Eq.(17) we found that, at the 90% confidence level, $\Delta N/N \gtrsim 1$ (*i.e.* more than 100% uncertainty!) for $T \gtrsim 9.7$, 3.7 and 1.9keV at $z = 0$, 0.5, and 1, respectively.

We plot in Fig.1 the region spanned by the predicted TF, for both $\Omega_0 = 1$ and $\Omega_0 = 0.2$, once the statistical uncertainty (at the 90% c.l.) on the best fit values for both n and $\delta_v b$ are taken into account. Thus, if we write the TF as $N(T, z|\sigma_8, n)$, the uncertainty region is bounded by $N(T, z|\sigma_8 + \Delta\sigma_8, n - \Delta n)$ and $N(T, z|\sigma_8 - \Delta\sigma_8, n + \Delta n)$. Here $\Delta\sigma_8$ and Δn are the uncertainties on σ_8 and n derived, at a given confidence level, form the fit to the local TF (see Table 1).

It is evident the large degree of overlap of the two regions in the $3 \div 10$ keV range, up to redshift of order of unity. This overlap would have been even more substantial if we had taken into account the 10% uncertainty in the $M - T$ relation implied by the Evrard simulations. This is why we believe that the H&M conclusion that the TF evolution can test, at least for PLPS models and in the framework of the described procedure, the geometry of the universe has to be taken with care. As found by Eke *et al.* , the TF evolution is basically the same in low density, open and vacuum dominated models up to $z \approx 0.5$.

The behaviour of the uncertainty region can be completely understood with the help

of Eq.(17) above. There is a temperature such that $\Delta N/N = 0$ even if $\Delta b/b \neq 0$: this explains the region of minimum (or even zero) uncertainty, which depends upon Ω_0, z, and $\delta_v b$. Obviously, the region of maximum uncertainty is around the cutoff temperature, T_c: small variations in the parameters result in huge TF variations at $T \gtrsim T_c$. The uncertainty region shown in Fig.1 spans several orders of magnitude: this is simply an artifact of the logarithmic scale: having uncertainties of $\approx 100\%$ obviously means $0 \lesssim N(T,z) \lesssim 2\overline{N}$, where $\overline{N}$ is the TF obtained with the best fit values.

We reach similar conclusions for the CDM scenario. With the best fit values for h and $\delta_v b$ (at given Ω_0; see Table 2), we conclude that the TF evolution is again quite different in low-density ($\Omega_0 = 0.3$, vacuum dominated, say) and flat models. The former is the same cosmological model considered by Eke *et al.* However, the 90% confidence level regions obtained by considering the statistical uncertainties on the best fit values are substantially overlapped in the $3 \div 10$ keV up to $z \sim 0.5$ (see Fig.2). The uncertainty region in Fig.2 is evaluated as in the PLPS case. Moreover, as the flat CDM model does not provide a reasonable fit to the local TF, we show the uncertainty regions for two CDM+Λ models, with $\Omega_0 = 0.7$ and 0.3, respectively. We obtain very similar results for the cluster redshift distributions (see Fig.3).

So, from one hand we agree with the conclusion of Eke *et al.* that "even at $z = 0.33$, these (temperature) distributions depend very strongly on Ω_0". On the other hand, normalizing to the local TF yields large uncertainties in the theoretical predictions of the cluster TF evolution. Because of these uncertainties, we do not believe that measurements of the cluster TF at high z can yet provide a good estimate for Ω_0. We want also to stress that assuming $\beta \simeq 1$, only to reproduce the Eke *et al.* T_{15} value, would have increased the degree of overlap of the 90 % c.l. regions of Fig.2 even at $z \simeq 1$.

6. Discussion and conclusions

We have shown that the present data on the local TF and the lack of a self-consistent theory for cluster formation and evolution strongly weaken the predictive power of the theory. In fact, as discussed in Section 3, normalizing the models parameters (mainly $\delta_v b$) to the local TF determines the degree and kind of the TF evolution. Moreover, just because of the quality of the available data, the fit can not be so precise to have a clear cut distinction between the predictions of low-density and flat models. This is why we believe that, although in principle possible, any test of the geometry of the universe using the cluster TF at intermediate redshifts ($z \lesssim 0.5$) can not be, at the moment, very precise. We want to stress that our quoted uncertainties on the best fit values are systematically lower than those quoted by other authors. The uncertainty regions of Figs. 1 and 2 would have been broader if we had: *i)* included a 10% scatter in the $M - T$ relation derived from the Evrard simulations; *ii)* used the best fit model, *i.e.* CDM+Λ with $\Omega_0 = 0.4$; *iii)* included other systematic uncertainties related to measurements of the cluster temperatures and to the catalogue incompleteness (see Eke *et al.*); *iv)* considered for h the actual uncertainties derived from the χ^2 analysis, without limiting the allowed h values to the standard interval $0.5 \leq h \leq 1$; *v)* used the Eke *et al.* value of $T_{15} = 7.8$ keV, instead of our $T_{15} = 5.8$ keV.

In any case, even if no uncertainties were included (*i.e.* all the parameters such as $b\delta_v$ and the others were known with very high precision) we find that in order to reject the hypothesis of an $\Omega_0 = 1$, CDM universe (at 90% confidence level), we should observe $\gtrsim 16$, 7 keV clusters per steradian at $z \approx 0.3$. The intrinsic statistical uncertainty of the $N(T = 7\mathrm{keV}, z = 0.3)$ would be $\approx 2 \times \sqrt{N}$. Such a measurement would then provide $\Omega_0 \approx 0.7^{+0.3}_{-0.2}$, again, at the 90% confidence level. However, if one takes into accont the parameter uncertainties, this estimate weakens quite considerably (see Fig.4). In any case, in order to have a complete sample of high-z clusters with precise temperature determinations

we have to wait for the next generation of X-ray space experiments, featuring good energy resolution ($E/\Delta E \gtrsim 50$), large photon collecting areas $A_{eff} \gtrsim 2 \times 10^2 cm^2 (@7keV)$ and a large sky coverage ($\gtrsim$ a few sr). The planned X-ray missions of the next decade (such as AXAF, XMM, SPECTRUM-X-γ, ABRIXAS, ASTRO-E and HTSX) could yield such detailed informations on high-z clusters provided that a total observing time $\gtrsim$ a few $10^6 s$ will be devoted to such studies.

Acknowledgements We are indebted with the Referee for helpful comments and suggestions that improved the presentation of our results.

REFERENCES

Banday, A.J. *et al.* 1994, ApJ, 436, L99

Barbosa, D., Bartlett, J.G., Blanchard, A., and Oukbir, J. 1996, A&A, 314, 13

Bartlett, J.G., and Silk, J., 1993, ApJ, 407, L45

Bond, J.R. et al. 1991, ApJ, 379, 440

Cavaliere, A. and Colafrancesco, S. 1988, ApJ, 331, 660

Cirimele, Nesci, R. and Trevese, D. 1997, ApJ, 475, 11

Colafrancesco, S., Mazzotta, P., Rephaeli, Y., and Vittorio, N. 1997, ApJ, in press

de Bernardis, P., Balbi, A., De Gasperis, G., Melchiorri, A. and Vittorio, N. 1996, ApJ, in press

Ebeling, H. *et al.* 1997, preprint astro-ph/9701179

Edge, A.A., Stewart, G.C., Fabian, A.C., Arnaud, K.A., 1990, MNRAS, 245, 559

Eke, V.R., Cole, S., and Frenk, C.S. 1996, MNRAS, 282, 263

Evrard, A.E. 1991, in 'Clusters of Galaxies', Eds. M. Fitchett and W. Oegerle (Cambridge: Cambridge Univ. Press)

Gioia, I. *et al.* 1990, ApJ, 356, L35

Gorski, K.M., *et al.* 1994, ApJ, 430, L89

Jones, C. & Forman, W. 1992, in *Clusters and Superclusters of Galaxies*, A.C. Fabian *et al.* eds., (Cambridge: Cambridge University Press), p.49

Hattori, M. and Matsuzawa, H. 1995, A&A, 300, 637

Henriksen, M., and Mushotsky, R. 1985, ApJ, 292, 441

Henry, J.P. and Arnaud, K.A., 1991, ApJ, 372, 410

Henry, J.P., Gioia, I.M., Maccacaro, T., Morris, S.L., Stocke, J.T., & Wolter, A. 1992, ApJ, 386, 408

Kaiser, N. 1991, ApJ, 383, 104

Kowalski, M.P. et al., 1984, ApJS., 56, 403

Nichol, R.C. *et al.* 1996, preprint astro-ph/9611182

Oukbir, J. and Blanchard, A. 1992, A&A, 262, L21

Oukbir, J. and Blanchard, A. 1997, A&A, 317, 1

Peebles, P.J. (1980), *'The Large Scale Structure of the Universe'*, (Princeton: Princeton University Press).

Press, W.H. and Schechter, P. 1974, ApJ, 187, 425

Romer, A.K. *et al.* 1997, preprint astro-ph/9701233

Smoot, G.F., Bennett, C.L., Kogut, A., Wright, E.L., Aymon, J. et al. 1992, ApJ, 396, L1

Viana, P.T.P. and Liddle, A.R. 1996, MNRAS, 281, 323

White, S.D.M., et al. 1993, Nature, 366, 429

White, S.D.M.,Efstathiou, G. and Frenk, C.S. 1993, MNRAS, 262, 1023

This manuscript was prepared with the AAS LaTeX macros v4.0.

Fig. 1.— Uncertainty regions in the TF predictions for PLPS models at redshifts $z = 0.3$ (upper panel), $z = 0.5$ (middle panel) and $z = 1$ (lower panel). The shaded (dashed for $\Omega_0 = 1$ and dotted for $\Omega_0 = 0.2$) areas are drawn considering the uncertainties (at 90% confidence level) in the model parameters (see text).

Fig. 2.— Same as Figure 1 but for two low-density, vacuum dominated CDM models: $\Omega_0 = 0.3$ (dotted shaded area) and $\Omega_0 = 0.7$ (dashed shaded area).

Fig. 3.— Uncertainty regions for the redshift distributions of clusters with $T > 3keV$ (panel a) and $T > 7keV$ (panel b) as predicted by the same models of Figure 2.

Fig. 4.— Cluster TF for the flat CDM model ($\Omega_0 = 1$, $h = 0.5$, $\delta_v b = 2.5$: continuous line) and for two CDM+Λ models ($\Omega_0 = 0.7$, $h = 0.5$, $\delta_v b = 2.05$: dashed line; $\Omega_0 = 0.5$, $h = 0.6$, $\delta_v b = 1.78$: dot-dashed line). The shaded area shows the uncertainty region for a low density ($\Omega_0 = 0.3$) CDM+Λ) model, once the uncertainties on $\delta_v b$ and h are taken into account.

Table 1. Fitting parameters for the TF: PLPS models.

Ω_0	n	$b\delta_v$	χ^2_{min}
1	$-1.8^{+0.41(+0.75)}_{-0.44(-0.90)}$	$2.8^{+0.17(+0.24)}_{-0.10(-0.25)}$	0.66
0.2	$-0.9^{+0.63(+1.25)}_{-0.72(-1.13)}$	$1.1^{+0.23(+0.38)}_{-0.23(-0.41)}$	0.59
0.1	$-0.5^{+0.53(+1.19)}_{-0.44(-1.00)}$	$0.6^{+0.11(+0.31)}_{-0.13(-0.27)}$	0.74

Table 2. Fitting parameters for the TF: CDM models.

Model	Ω_0	$b\delta_v$	h	χ^2_{min}
SCDM	1	$2.5^{+0.0(+0.03)}_{-0.0(-0.03)}$	$0.5^{+0.0(+0.02)}_{-0.0(-0.0)}$	1.84
CDM+Λ	0.4	$1.6^{+0.05(+0.14)}_{-0.13(-0.19)}$	$0.6^{+0.4(+0.4)}_{-0.1(-0.1)}$	0.58
CDM+Λ	0.3	$1.4^{+0.11(+0.23)}_{-0.19(-0.25)}$	$0.7^{+0.3(+0.3)}_{-0.2(-0.2)}$	0.74
CDM	0.3	$1.5^{+0.10(+0.23)}_{-0.20(-0.25)}$	$0.65^{+0.35(+0.35)}_{-0.15(-0.15)}$	0.72

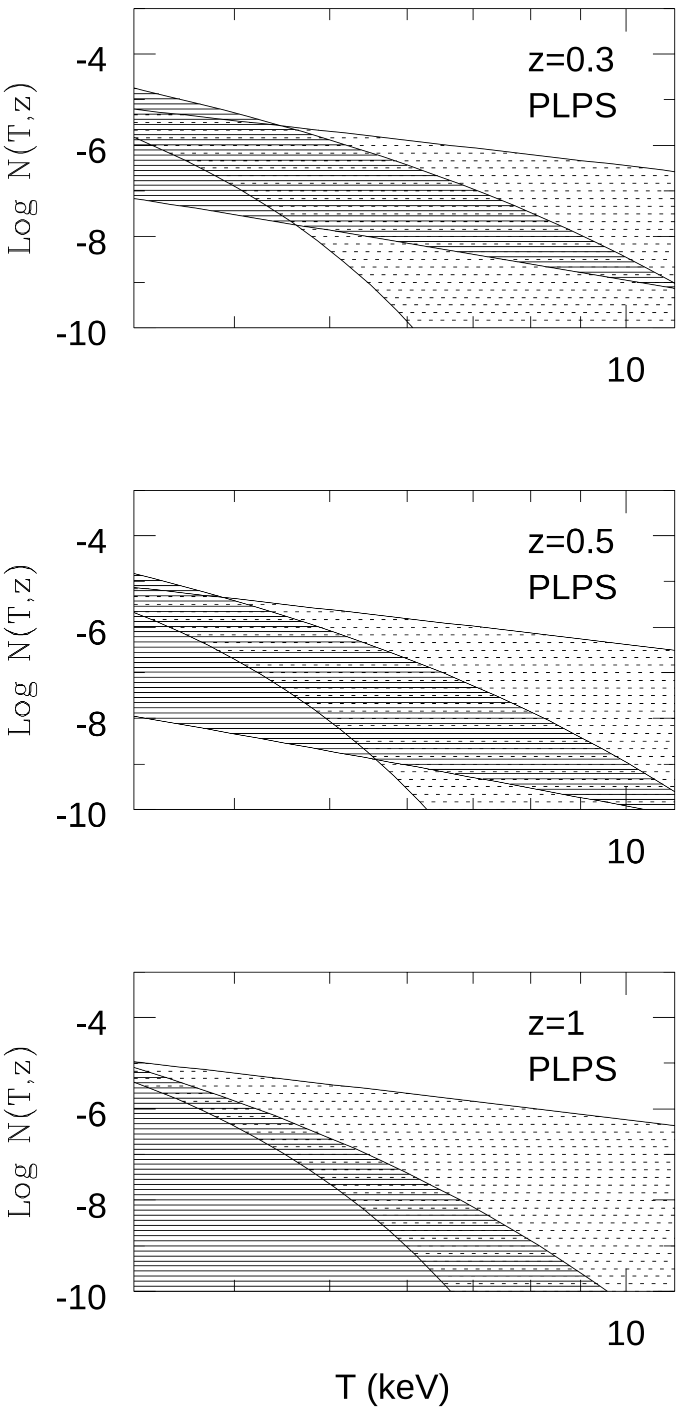


Fig.1

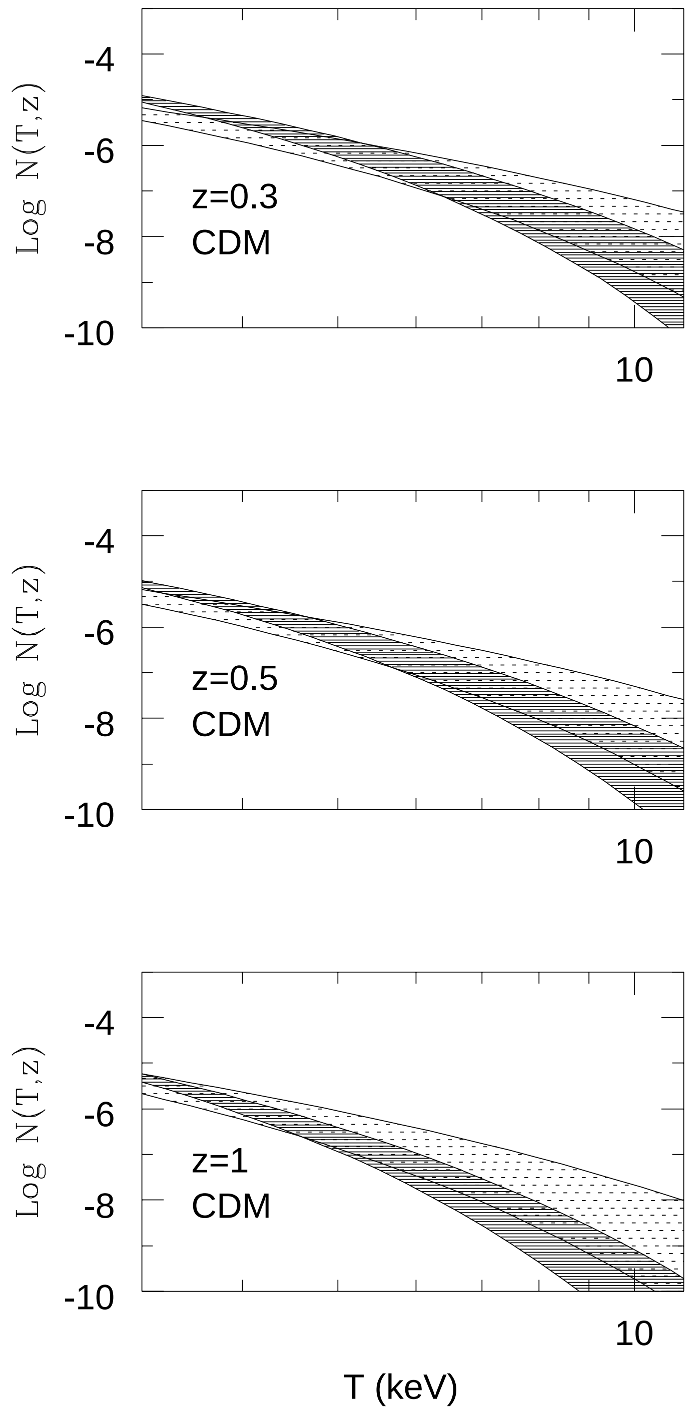
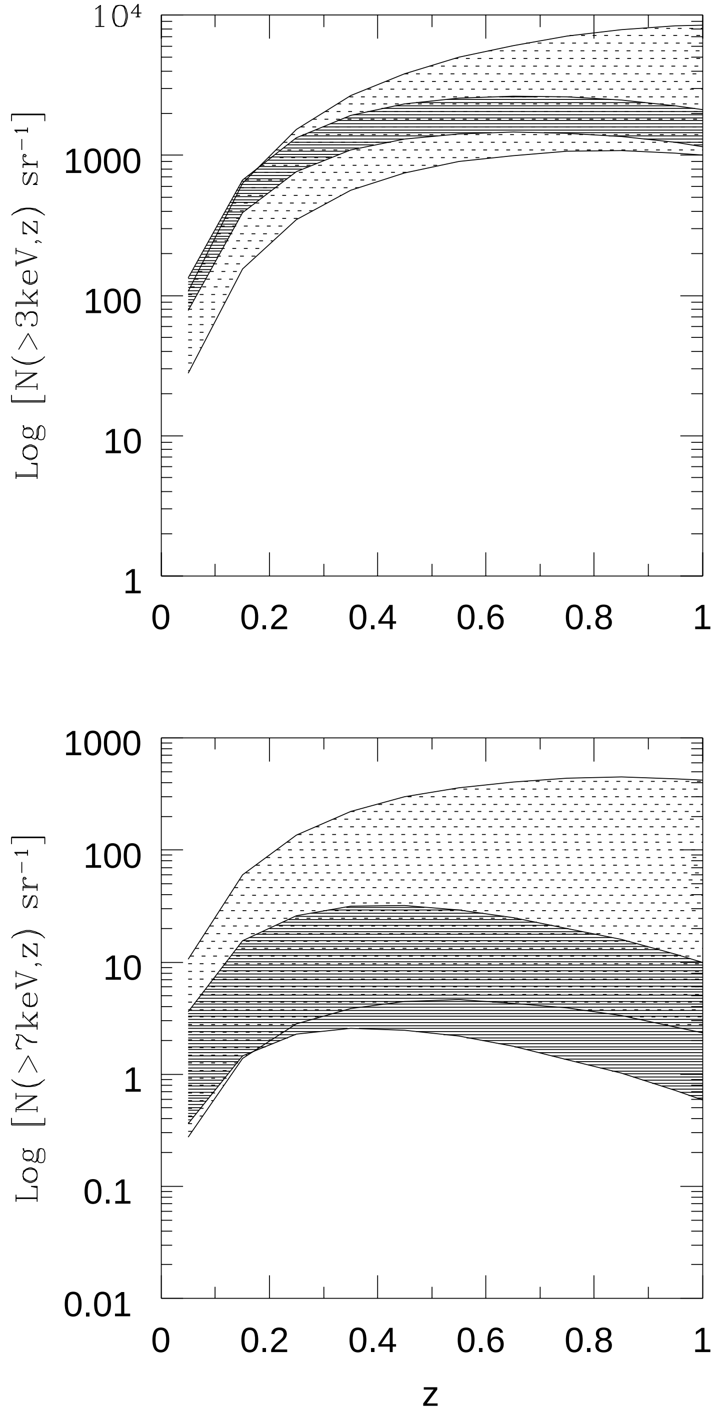

Log N(T,z)
-4
-6
-8
-10
z=0.3
CDM
10
Log N(T,z)
-4
-6
-8
-10
z=0.5
CDM
10
Log N(T,z)
-4
-6
-8
-10
z=1
CDM
10
T (keV)

Fig.2



10^4
1000
100
10
1
0
0.2
0.4
0.6
0.8
1
Log [N(>3keV,z) sr^{-1}]
1000
100
10
1
0.1
0.01
0
0.2
0.4
0.6
0.8
1
Log [N(>7keV,z) sr^{-1}]
z

Fig.3

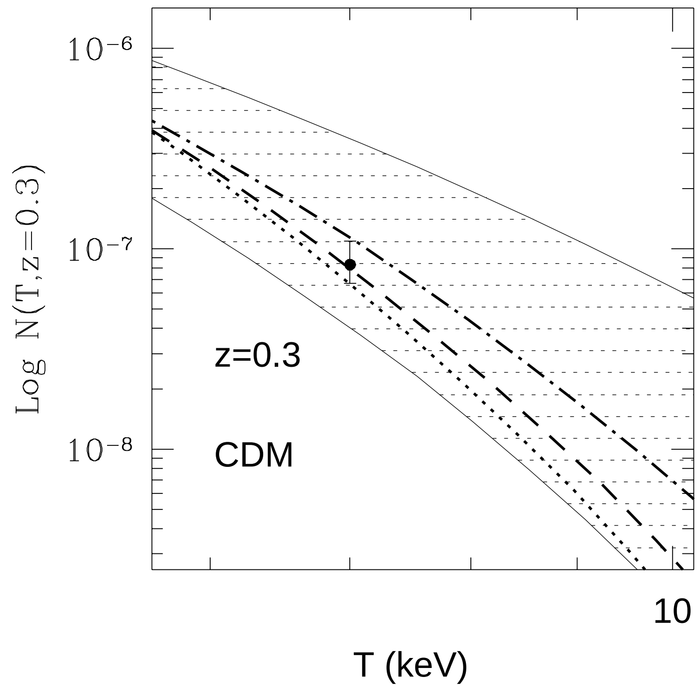


Fig.4